Theory of Simple Afterglows

Dynamics and Radiation of Relativistic Synchrotron Shocks

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Introduction and Plan

Here I concentrate on basics: spherical blast waves and their synchrotron emission. See later for jets and other complexities.

- √ Shocks and jump conditions (Blandford & McKee 1976)
- √ Dynamics
- √ Thermodynamics/Fields
- √ Radiation

Shocks and jump conditions

Like Rankine-Hugoniot for normal shocks, relativistic shocks have jump conditions (Taub). In the ultrarelativistic ($\gamma \gg 1$), strong shock ($\mathcal{M} \gg 1$ or $P_{\mathrm{after}} \gg P_{\mathrm{before}}$) limit, these are:

$$\begin{aligned} n' &= 4\gamma n && (1) \\ U' &= 4\gamma n \cdot (\gamma - 1) m_{\mathrm{p}} c^2 && (2) \end{aligned}$$

(See Landau & Lifshitz vol.6 or BM76)

Note1: primed quantities are in the restframe of the blast wave

Note2: henceforth, we neglect flow structure behind shock: all shocked gas is in uniform slab behind shock (see BM76 for better way). Right there, we give up getting answers to better than factor 2.

Shell equation of motion 1

This is often incorrectly done in literature, so beware.

First: crude version, adiabatic. Initially, shell has Lorentz factor γ_0 and mass M_0, so $E_0 = \gamma_0 M_0 c^2$.

A swept-up and shocked mass m has thermal energy $\gamma m c^2$ in the shock frame (jump condition), and thus $\gamma^2 m c^2$ in our frame. Equating the two, we get two results:

- ✓ Deceleration starts in earnest when shocked gas has similar energy to initial: $E_0 = \gamma_0 M_0 c^2 \simeq \gamma_0^2 m c^2 \Longrightarrow m = M_0/\gamma_0$.
- ✓ Once $m \gg M_0$, we have $E_0 = \gamma^2 m c^2 \Longrightarrow \gamma \propto m^{-1/2}$.

Shell equation of motion 2

More precisely, and adding energy loss, E_{k} of the shocked shell in our frame, and the loss when sweeping up mass $\mathrm{d}m$ are (BM76, Panaitescu and Mészáros 1998):

$$E_{\mathrm{k}} = (\gamma - 1)(M_0 + m)c^2 + (1 - \epsilon)\gamma U' \tag{3}$$

$$\mathrm{d}E_{\mathrm{rad}} = \epsilon\gamma(\gamma - 1)c^2\mathrm{d}m \tag{4}$$

(so $\epsilon = 0$ is adiabatic, $\epsilon = 1$ is fully radiative; only the former is treated consistently in literature.)

Combine:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}m} = -\frac{\gamma^2 - 1}{M_0 + \epsilon m + 2(1 - \epsilon)\gamma m} \tag{5}$$

Shell equation of motion 3

Radiative, $\epsilon = 1$.

Note: all light from narrow boundary layer after the shock. This case is often erroneously used for $\epsilon = 0$ as well.

$$\frac{\mathrm{d}\gamma}{\mathrm{d}m} = -\frac{\gamma^2 - 1}{M_0 + m} \Longrightarrow \tag{6}$$

$$\left(\frac{\gamma - 1}{\gamma + 1}\right)\left(\frac{\gamma_0 + 1}{\gamma_0 - 1}\right) = \left(\frac{M_0 + m_0}{M_0 + m}\right)^2 \tag{7}$$

Two limits:

- ✓ the shell comes to a stop within a few times M_0/γ_0, so expand for $m \ll M_0$: $\gamma \propto m^{-1}$ ($\gamma mc = cst.$) (!!)
- ✓ for $m \gg M_0$, non-relativistic. Then $\beta \propto m^{-1}$ (snowplow).

Shell equation of motion 4

Adiabatic, $\epsilon = 0$.

$$\frac{\mathrm{d}\gamma}{\mathrm{d}m} = -\frac{\gamma^2 - 1}{M_0 + 2\gamma m} \Longrightarrow \tag{8}$$

$$\gamma M_0 + (\gamma^2 - 1)m = cst. \tag{9}$$

Rearrange this a bit:

$$(\gamma - 1)M_0 c^2 + (\gamma - 1)mc^2 + \gamma(\gamma - 1)mc^2 = E_{\mathrm{k}0} \tag{10}$$

Two limits:

- ✓ $m \gg M_0$: third term dominates, so $\gamma \propto m^{-1/2}$.
- ✓ $\gamma \simeq 1$, $\gamma^2 \simeq 1 + \frac{1}{2}\beta^2$: then $\frac{1}{2}mv^2 = E_{\mathrm{k}0}$: Sedov-Taylor.

Kinematic relation

Funny thing about time between explosion rest frame and our frame. We measure time as the arrival time difference between light emitted at explosion and light emitted from radius r by the blast wave:

$$t(\equiv t_{\rm obs}) = \frac{r}{\beta c} - \frac{r}{c} = \frac{r}{2\gamma^2 c}\frac{2}{\beta(1+\beta)} \qquad (11)$$

If γ varies, then still OK differentially: $\mathrm{d}t = \frac{\mathrm{d}r}{2\gamma^2 c}$

This leads to counterintuitive $r(t)$, e.g., in uniform medium we have $\gamma \propto m^{-1/2} \propto r^{-3/2}$, so

$$\mathrm{d}t \propto \gamma^{-2}\mathrm{d}r \propto r^3 \mathrm{d}r \Longrightarrow r \propto t^{1/4} \qquad (12)$$

Aside: internal shocks

With all these Lorentz factors, how can GRB prompt emission fluctuations measure engine behaviour?

Two shells, emitted $\Delta t_{\rm em}$ apart, with Lorentz factors $\gamma_{1,2}$. Collision at $r = c\Delta t_{\rm em}\frac{\beta_1\beta_2}{\beta_2-\beta_1}$.

If $\gamma_2 \sim 2\gamma_1$ (to get good radiation efficiency), then $r \sim \gamma_1^2 c\Delta t_{\rm em}$.

Finally, $t_{\rm obs} \sim \frac{r}{\gamma_1^2 c} \sim \Delta t_{\rm em}$ (!!)

A few numbers – Adiabatic

Time to start decelerating in uniform medium of density n:

$$m(r_{\rm dec})\gamma_0^2 c^2 = E_0 \Longrightarrow \tag{13}$$

$$r_{\rm dec} = \left(\frac{3E_0}{4\pi\gamma_0^2 n m_{\rm p} c^2}\right)^{1/3} = 1.8\times 10^{16}\left(\frac{E_{52}}{n}\right)^{1/3}\gamma_{0,300}^{-2/3}\,{\rm cm} \tag{14}$$

$$t_{\rm dec} = \frac{r_{\rm dec}}{2\gamma_0^2 c} = 3.4(E_{52}/n)^{1/3}\gamma_{0,300}^{-8/3}\,{\rm s} \tag{15}$$

Note strong dependence on γ_0 and weak dependence on E and n.

The relativistic phase ends when $E_{k0} = mc^2$, or

$$t_{\rm nr} = \left(\frac{3E}{4\pi n m_{\rm p} c^5}\right)^{1/3} \simeq 0.5\,{\rm yr}\left(\frac{E_{51}}{n}\right)^{1/3} \tag{16}$$

Radiation

At $r_{\rm dec}$, shell density and optical depth small: synchrotron radiation.

Get magnetic field and relativistic electrons from parametrized ignorance:

$U'_B = \epsilon_B U'; \qquad U'_e = \epsilon_e U'$

Furthermore, we assume the accelerated electrons have some minimum Lorentz factor $\gamma_{\rm m}$ and a power-law distribution above: $n(\gamma) \propto \gamma^{-p}$.

Result:

$$\begin{aligned} \gamma_{\rm m} &= k_1 \gamma \epsilon_e \frac{m_{\rm p}}{m_{\rm e}} && (17) \\ B' &= k_2 \gamma \epsilon_B^{1/2} && (18) \end{aligned}$$

Synchrotron properties

Roughly, we get the following synchrotron characteristics, with numbers put in for the spherical adiabatic case (see Rybicki & Lightman, Wijers & Galama 1999):

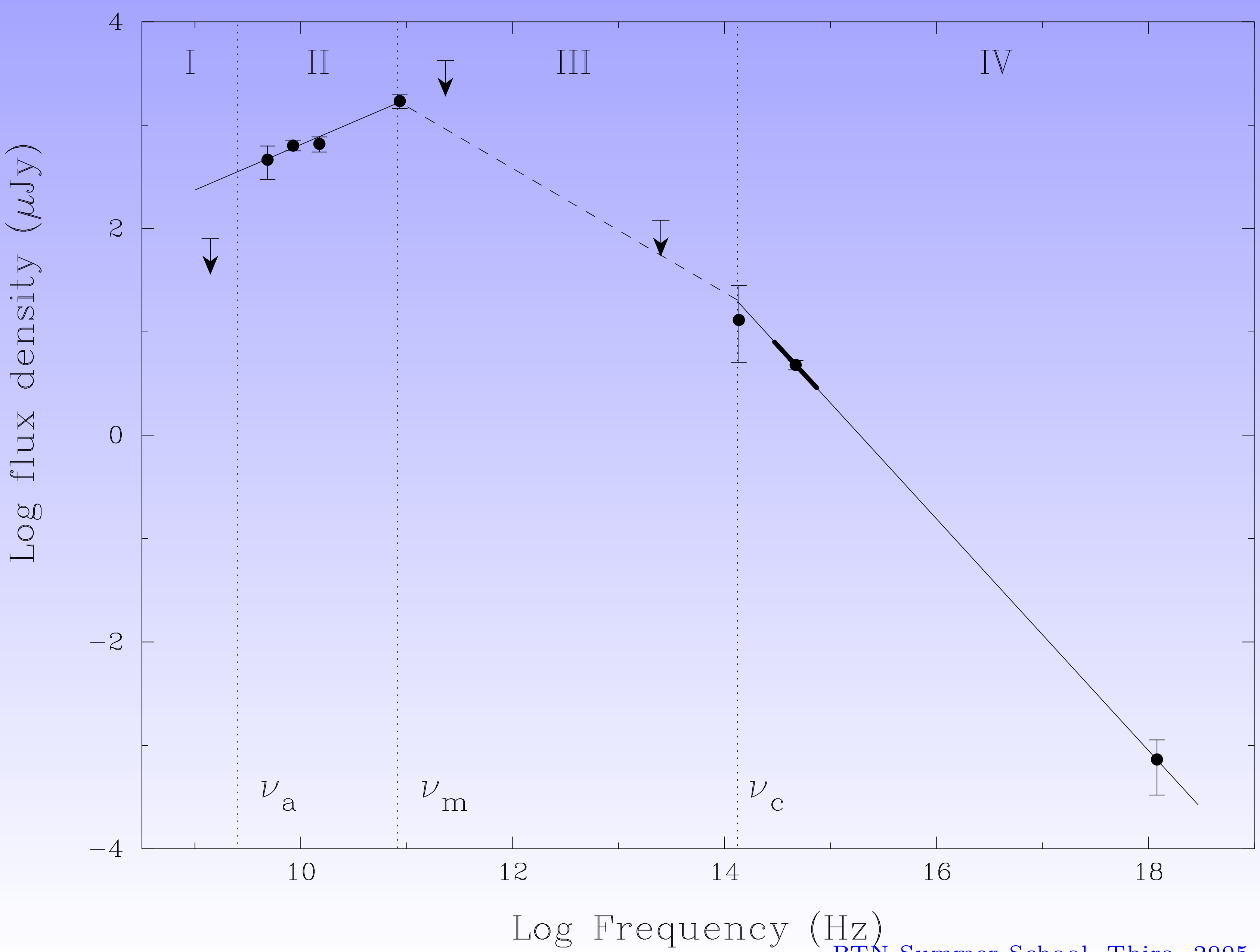
$$\nu_{\rm m} \propto \gamma_{\rm m}^2 \gamma B' \quad \sim 3 \times 10^{13}\,{\rm Hz}\; t_{\rm d}^{-3/2} \quad \epsilon_{{\rm e},-1}^2 \epsilon_{B,-2}^{1/2} E_{52} \tag{19}$$

$$\nu_{\rm c} \propto (\gamma t^2 B'^3)^{-1} \quad \sim 1 \times 10^{15}\,{\rm Hz}\; t_{\rm d}^{-1/2} \quad \epsilon_{B,-2}^{-3/2} E_{52}^{-1/2} n^{-1} \tag{20}$$

$$\nu_{\rm a} \quad \sim 1 \times 10^{9}\,{\rm Hz} \quad \epsilon_{{\rm e},-1}^{-1} \epsilon_{B,-2}^{1/5} E_{52}^{1/5} n^{3/5} \tag{21}$$

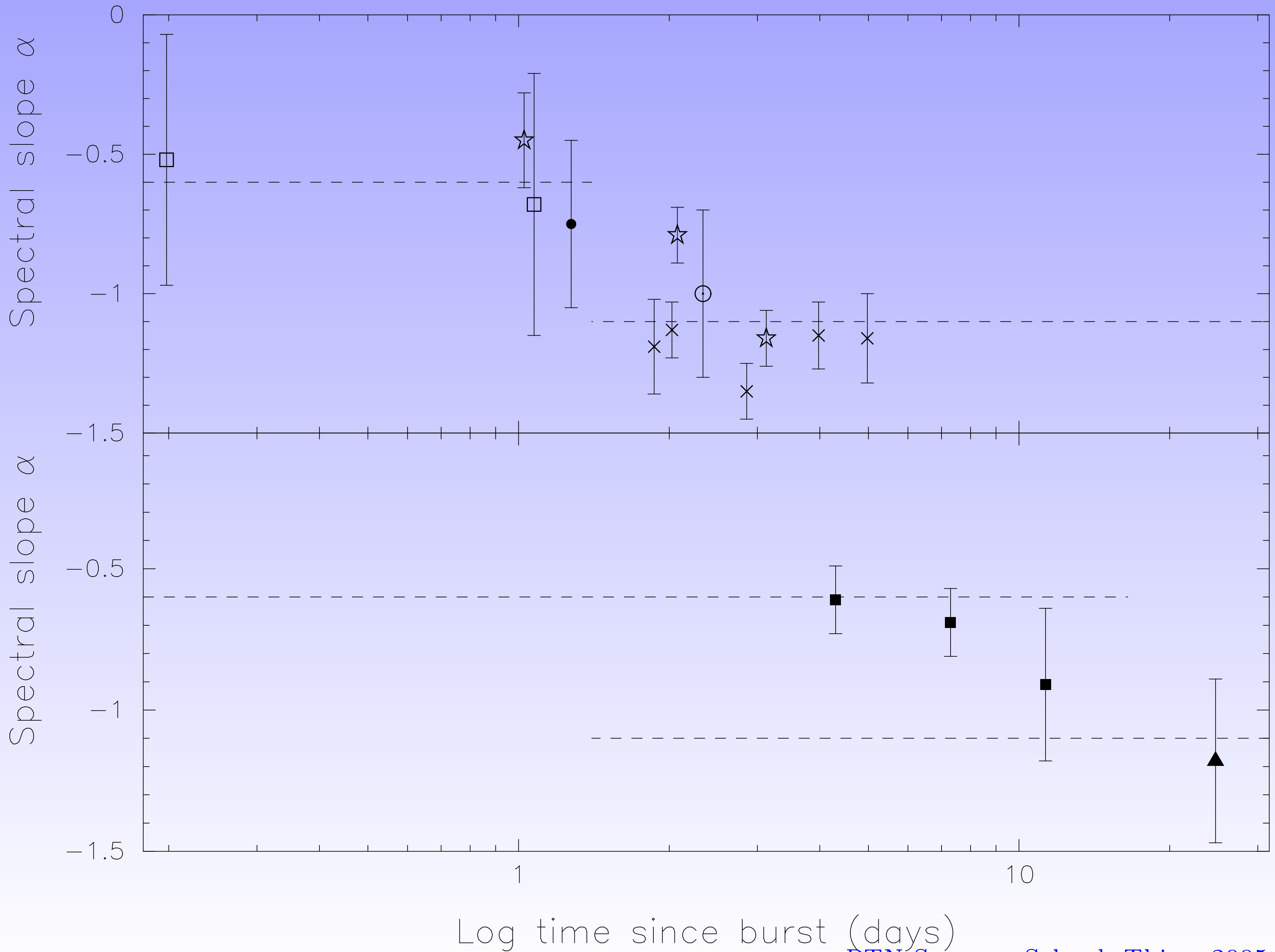
$$F_{\rm m} \propto \gamma m B' \quad \sim 1\,{\rm mJy} \quad \epsilon_{B,-2}^{1/2} E_{52} n^{1/2}\ (z=1) \tag{22}$$

Some data 1



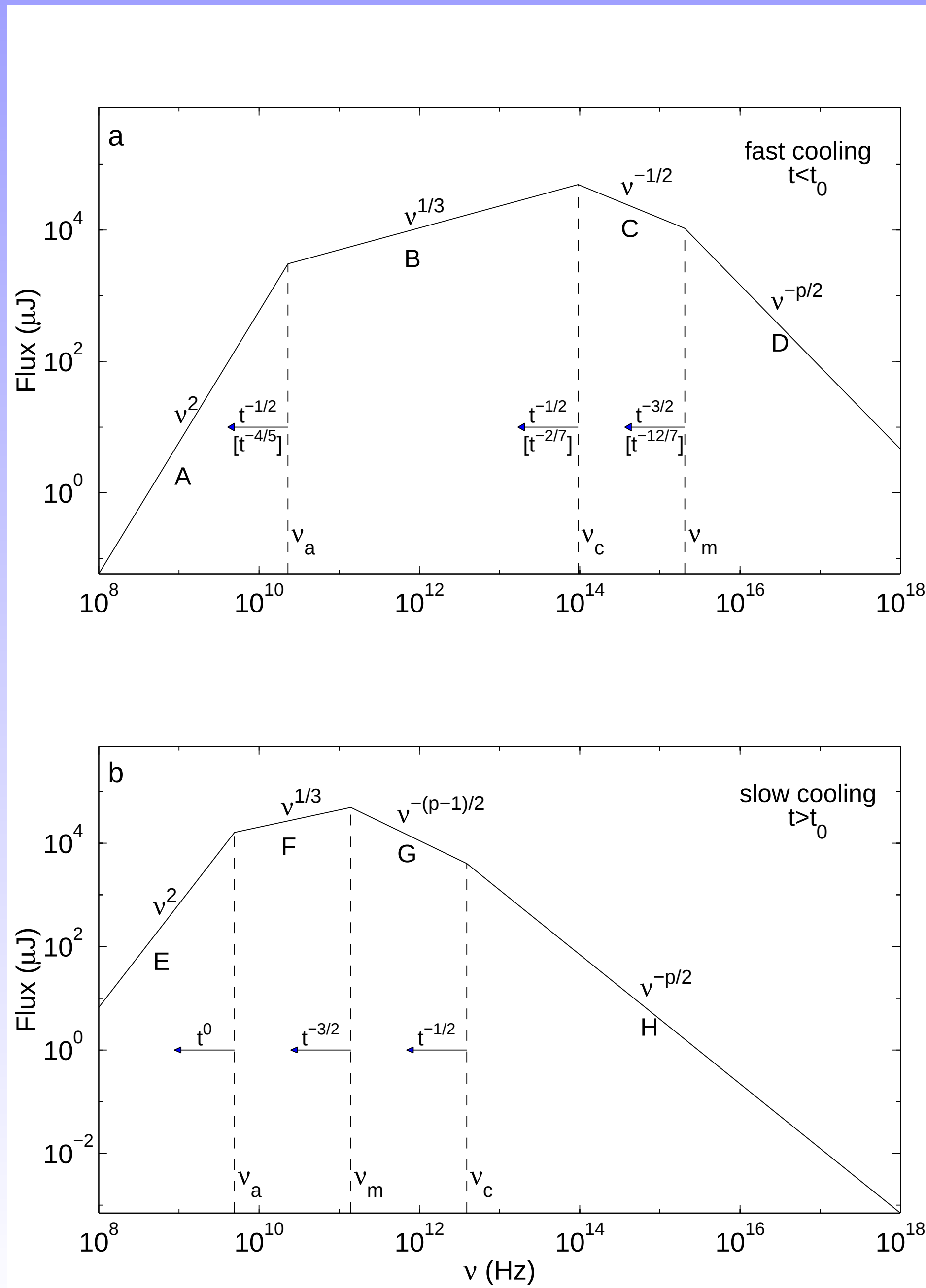
I
II
III
IV
ν_a
ν_m
ν_c
Log flux density (μJy)
Log Frequency (Hz)
4
2
0
−2
−4
10
12
14
16
18

Some data 2



0
−0.5
−1
−1.5
−0.5
−1
−1.5
Spectral slope α
Spectral slope α
1
10
Log time since burst (days)

More models



a
fast cooling
t<t0
ν^{1/3}
ν^{−1/2}
B
C
ν^{−p/2}
D
ν^2
A
t^{−1/2}
[t^{−4/5}]
t^{−1/2}
[t^{−2/7}]
t^{−3/2}
[t^{−12/7}]
νa
νc
νm
Flux (μJ)
b
slow cooling
t>t0
ν^{1/3}
ν^{−(p−1)/2}
F
G
ν^2
E
ν^{−p/2}
H
t^0
t^{−3/2}
t^{−1/2}
νa
νm
νc
Flux (μJ)
ν (Hz)

Inverse Compton?

Inverse Compton photons are $\gamma_{\rm m}^2$ times higher in energy, thus even at same energy output $\gamma_{\rm m}$ times lower in flux, mostly unimportant in observed spectrum, BUT

$$\tau_{\rm T} = N\sigma_{\rm T} \sim nr\sigma_{\rm T} \sim 10^0 \cdot 10^{17} \cdot 10^{-24} = 10^{-7} \quad (23)$$

$$\gamma_{\rm m} = \gamma\epsilon_{\rm e} m_{\rm p}/m_{\rm e} \sim 10 \cdot 10^{-1} \cdot 10^3 = 10^3 \quad (24)$$

(For spherical adiabatic case at 1 day; scales as $t^{-1/2}$.)

Compton $y \equiv \gamma_m^2 \tau \sim 0.1 - 1$ is possible $\Longrightarrow$ Inverse compton may influence early blastwave evolution and may give X-ray excess.

Other cooling processes: neutrinos, CR proton & neutron leaks, ...

A few basic references

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